LAN : link layer performance

RES 841

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Note : formulas are provided at the end of the sheet if necessary

Above the binary channel provided by the physical layer, the link layer implements several mechanisms in order to use more efficiently the available capacity, given parameters such as the bit-error-rate. The exercices below will have you manipulate classical link-layer metrics (capacity, delay, error probability, ...) and to illustrate the way standard parameters are set. Note that, in the industry, such analysis only provides hints on the parameters best values and the real parameters values (e.g. the MTU) are generally determined empirically.

1 Optimal frame size

Let us suppose that we have a transmission link that offers a capacity $D = 10\,Mbit/s$ and a bit-error probability $P_b = 10^{-6}$. On this link, we want to transmit a file whose size is $F = 100\,Mbit$. The link layer does not provide any error correction algorithm, therefore any single error will provoke the retransmission of the whole frame.

The file can be divided into multiple frames. Each frame is composed of the data itself, plus a header that represents $H = 40\,bytes$, i.e. $H = 320\,bits$. We want to study the influence of the frame size, noted L below, on the transmission performance.

1. Express P_f, the probability that an L bits long frame, to which we add H header bits, experiences at least one transmission error. This probability is a function of the bit error probability, P_b, supposed to be constant during the whole frame transmission.
2. Express N_{TX}, the expected number of tries to transmit the frame without error in function of P_f.
3. Express T_f, the total time required to achieve a correct transmission of the full file in function of F, L, H, D and P_b. You will consider that each frame is transmitted on average N_{TX} times and that F is a multiple of L (to avoid manipulating ceiling and floor functions).
4. The expression obtained at the previous question should be continuous and differentiable on $L \in \mathbb{R}^{+*}$. Express this function's derivative according to variable L.
5. Find the zeroes of this derivative and study its sign on $L \in \mathbb{R}^{+*}$. What can you conclude?
6. Express the optimal frame size in function of the parameters of the exercice. If you have a computer / calculator, you can find this value for the given figures.
7. Which parameters do (and do not) influence the optimal frame size? Comment this result.

2 Anticipation window

Let us not consider a perfect link (bit-error rate is equal to zero), whose capacity is D bit/s and whose propagation time is δ s. Propagation time represent the time necessary for a bit to go from one end of the link to the other end.

1. Express the time required for transmission of an L bits frame, without additional header, in function of D and δ.

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2. If the link-level protocol imposes a stop-and-wait acknowledgment strategy, express the time required to transmit a file os F bits, separated in frames of length L bits. You will consider that F is a multiple of L to avoid rounding problems. You will also consider that the acknowledgment size is negligible and that transmitting an acknowledgment only requires one propagation time.
3. Express the effective throughput as the ratio between the data volume transmitted and the time required for this transmission.
4. Compare this effective throughput for a file of size $F = 100 Mbit$, when the channel capacity is$D = 10\,Mbit/s$ in the cases $\delta = 5\,\mu s$ (wired transmission) and $\delta = 16\,ms$ (satellite link). You will consider a frame size of $L = 10000 bits$.
5. Comment the difference between both scenarios. Express the amount of data that the emitter could transmit while it waits. How many frames does this represent? Calculate, for the aforementioned figures, these values.
6. Which strategies could you imagine to improve this performance?

3 Formulas

3.1 Expected value of a geometric random variable with parameter $p < 1$

$$\sum_{i=1}^{+\infty} i \cdot p^{i-1} \cdot (1-p) = \frac{1}{1-p}$$

3.2 Derivatives

$$(a^x)' = (a^x) \cdot \ln(a)$$

$$(u \cdot v)' = u' \cdot v + u \cdot v'$$

$$\left(\frac{u}{v}\right)' = \frac{u' \cdot v - u \cdot v'}{v^2}$$

3.3 Quadratic equations

To find the zeros of $a \cdot x^2 + b \cdot x + c$, you need to compute the discriminant, whose expression is :

$$\Delta = b^2 - 4 \cdot a \cdot c$$

When $\Delta > 0$, the equation has two zeroes:

$$\frac{-b \pm \sqrt{\Delta}}{2 \cdot a}$$